

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-I & II EXAMINATION – WINTER 2024****Subject Code:3110019****Date:16-01-2025****Subject Name:Remedial Mathematics****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) Find $\lim_{x \rightarrow 2} \left(\frac{x^2 - 6x + 8}{x^2 - 4} \right)$ **03**
- (b) Find $\frac{dy}{dx}$ for the following **04**
- (1) $y = e^{\cot x}$ (2) $y = \log(\sin x^2)$
- (c) Show that the function $y = e^{3x} + e^{2x}$ is a solution of differential equation **07**
- $$\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$
- Q.2** (a) Find the equation of a line passing through the points (1, -3) & (5,6). **03**
- (b) Find the equation of a line passing through (-3,5) and perpendicular to the line through the points (2,5) and (-3,6). **04**
- (c) (1) If a line passing through the points (-2,5) and (4,9) is perpendicular to the line passing through the points (6,8) and (x, 20) then find the value of x. **03**
- (2) Find the angle between two lines **04**
- $$3x - y + 5 = 0 \text{ and } 2x - 4y + 11 = 0$$
- OR**
- (c) (1) Represent $\cos(x + y)$ in terms of sine and cosine and hence prove that **03**
- $$\cos(2x) = \cos^2 x - \sin^2 x$$
- (2) Find the distance of the point (5,2) from the line $3x - 4y + 13 = 0$ **04**
- Q.3** (a) Find the value of x, such that $\begin{vmatrix} x & 3 \\ 2 & x \end{vmatrix} = \begin{vmatrix} 14 & 4 \\ 3 & 5 \end{vmatrix}$ **03**
- (b) If $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$ then verify that $AB \neq BA$. **04**
- (c) Solve the following system of linear equations **07**
- $$x + y + 2z = 8, \quad -x - 2y + 3z = 1, \quad 3x - 7y + 4z = 10$$
- OR**
- Q.3** (a) Define (a) Identity matrix (b) Scalar matrix and (c) Diagonal matrix **03**
- (b) If $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 5 & 4 \end{bmatrix}$ then find $A \cdot B$ **04**
- (c) Solve the following system of linear equations **07**
- $$x + 2y - z = 1, \quad x + y + 2z = 9, \quad 2x + y - z = 2$$
- Q.4** (a) Determine the order and degree of following differential equations **03**
- (1) $\frac{d^2y}{dx^2} - 3 \left(\frac{dy}{dx} \right)^3 + 4y = 0$ (2) $\left(\frac{d^2y}{dx^2} \right)^3 - \left(\frac{d^3y}{dx^3} \right)^{\frac{1}{3}} - y = 0$
- (b) Solve : $\frac{dy}{dx} + \frac{y}{x} = x^2$ **04**
- (c) (1) Solve : $x^2 dy = y(x^2 + x + 1)dx$ **03**
- (2) If $y = xe^{3x}$ find $\frac{d^2y}{dx^2}$ **04**

OR

- Q.4 (a)** Evaluate : $\lim_{x \rightarrow 1} \left(\frac{x^{100} - 1}{x - 1} \right)$ **03**
- (b)** Solve : $\frac{dy}{dx} + 3y = e^{-x}$ **04**
- (c)** (1) If $y = \frac{e^x}{x^2 + 1}$ then find $\frac{dy}{dx}$ **03**
- (2) Solve : $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$ **04**

- Q.5 (a)** Evaluate: **03**

$$\int \left(\frac{x^2 - 7x + 10}{x} \right) dx$$

- (b)** Evaluate: **04**

$$\int \left(\frac{e^{2x}}{1 + e^{2x}} \right) dx$$

- (c)** Evaluate the following integrations **03**

(1)

$$\int (2e^{3x} + \sin 2x + 3) dx$$

(2)

04

$$\int_0^2 \frac{1}{(x + 1)(x + 2)} dx$$

OR

- Q.5 (a)** Evaluate: **03**

$$\int \left(\frac{1}{x^3} + \frac{1}{x^2} \right) dx$$

- (b)** Evaluate: **04**

$$\int \left(\frac{e^{\tan^{-1} x}}{1 + x^2} \right) dx$$

- (c)** Evaluate the integration **03**

(1)

$$\int \sin 3x \cos 2x dx$$

(2)

04

$$\int_2^3 \left(\frac{1}{x^2 - 1} \right) dx$$
