

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-I (NEW) EXAMINATION – WINTER 2023

Subject Code:3110019

Date:24-01-2024

Subject Name:Remedial Mathematics

Time:02:30 PM TO 05:00 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1 (a)** Define exponential function and logarithmic function 03
- (b)** Solve the following system by matrix method 04
 $2x + 5y = 1$
 $3x + 2y = 7$
- (c)** (i) Find $\frac{dy}{dx}$ if $y = \frac{x^5 - \cos x}{\sin x}$ 03
(ii) Find the limits: (a) $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x + 99}$ (b) $\lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$ 04
- Q.2 (a)** Evaluate the determinant $A = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 2 & 4 \\ 0 & 1 & -1 \end{vmatrix}$ 03
- (b)** Find $A.B$ if $A = \begin{bmatrix} 5 & 8 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & 0 \end{bmatrix}$ 04
- (c)** (i) Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = 100$ 05
(ii) Derive $\sin 2x = 2 \sin x \cdot \cos x$ 02
- OR**
- (c)** (i) Find $\frac{dy}{dx}$, if $x = a \cos \theta, y = a \sin \theta$ 05
(ii) Derive the trigonometry identity: $\cos 2x = \cos^2 x - \sin^2 x$ 02
- Q.3 (a)** State the Quotient (division) rule of derivative and compute the derivative of $f(x) = \frac{\sin x}{\cos x}$ 03
- (b)** Find the distance (i) of the point $(3, -1)$ from the line $3x - 4y - 26 = 0$. 04
(ii) between two parallel lines: $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.
- (c)** (i) Define the matrices (a) Identity matrix (b) Scalar matrix 03
(c) Diagonal matrix.
(ii) Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$ 04

OR

- Q.3 (a)** State the product rule of derivative and find the derivative of $f(x) = x + 3x^2 + 5x^3 + 7x^4 + 9x^5$ at $x = 1$. **03**
- (b)** Find the point on the x-axis which is at equidistance from the point (7,6) and (3,4). **04**
- (c)** (i) If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ then find the value of x . **03**
- (ii) If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line. **04**

- Q.4 (a)** Determine the order of the following differential equations **03**

(i) $\frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^3 + 2y = 0$ (ii) $\left(\frac{d^4y}{dx^4}\right)^2 + \left(\frac{d^2y}{dx^2}\right) + \sin y = 0$

(iii) $(y'')^2 + 3(y')^5 - y = 0$

- (b)** Solve the differential equations $x \frac{dy}{dx} + 2y = x^2$ **04**
- (c)** Solve the following equations: **03**
- (i) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ **03**
- (ii) $xcos\left(\frac{y}{x}\right) \frac{dy}{dx} = ycos\left(\frac{y}{x}\right) + x$ **04**

OR

- Q.4 (a)** Find all points of discontinuity for the function given by $f(x) = \begin{cases} x+2 & x < 1 \\ 2 & x = 1 \\ x-2 & x > 1 \end{cases}$ **03**

- (b)** If $y = xcosx$ then find $\frac{d^2y}{dx^2}$ **04**
- (c)** Solve the following equations: **03**
- (i) $xdy = (2x^2 + 1)dx$ **03**
- (ii) $x \frac{dy}{dx} + 2y = x^2$ ($x \neq 0$) **04**

- Q.5 (a)** Find $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ **03**

- (b)** Evaluate the integrals: (i) $\int \frac{dx}{x^2-81}$ (ii) $\int \frac{x^2-1}{x} dx$ **04**

- (c)** (i) Evaluate $\int_1^2 \frac{1}{(x+1)(x+2)} dx$ **03**
- (ii) Find $\int \frac{1}{x^3-6x+13} dx$ **04**

OR

- Q.5 (a)** State the integration by parts and find $\int x e^x dx$ **03**
- (b)** Find $\int \frac{\sin x}{(1+\cos x)^2} dx$ **04**
- (c)** (i) Evaluate $\int_0^{\frac{\pi}{2}} \cos x \cdot e^{\sin x} dx$ **03**
- (ii) Find $\int \frac{2x}{x^2+3x+2} dx$ **04**

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