Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III EXAMINATION – SUMMER 2025

Subject Code:3130005 **Date:13-06-2025**

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Find real and Imaginary parts of $z = \frac{3i}{2+3i}$.	**3**
	(b)	Find z, if $Arg\ (z+1) = \frac{\pi}{6}$ and $Arg\ (z-1) = \frac{2\pi}{3}$.	**4**
	(c)	Determine an analytic function whose real part is $u(x,y) = e^{2x}(x\cos 2y - y\sin 2y)$.	**7**
Q.2	**(a)**	Evaluate $\lim_{Z\to i} \frac{Z-i}{Z^2+1}$.	**3**
	(b)	Find $\log(1-i)$ and $\log(1+i)$.	**4**
	(c)	Find the bilinear transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.	**7**
		OR	
	(c)	Verify that $u = 2x - x^3 + 3xy^2$ is harmonic in the whole complex plane and finds it's harmonic conjugate function $v(x,y)$.	**7**
Q.3	**(a)**	Evaluate $\int_C \frac{e^z}{(z-1)(z-3)}\, dz$ counter clockwise around the circle $\lvert z\rvert = 2$.	**3**
	(b)	Determine the residues of the function $f(z) = \frac{z+1}{(z^2-16)(z+2)}$ at each of its poles.	**4**
	(c)	Evaluate $\int_C Re\ (z^2)\ dz$, where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clockwise wise direction.	**7**
		OR	
Q.3	**(a)**	Classify the poles of $(z) = \frac{1}{z^2-z^6}$.	**3**
	(b)	Find the Laurent's series of the function $f(z) = \frac{1}{(z+2)(z+4)}$ for the region $2 < \lvert z\rvert < 4$.	**4**
	(c)	Using Residue theorem evaluate the integral $\int_0^{\pi} \frac{1}{17-8\cos\theta}\, d\theta$.	**7**
Q.4	**(a)**	Determine the residues of the function $f(z) = \frac{3z-4}{z(z-1)(z-2)}$ at each of its poles.	**3**
	(b)	Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \sin(3x+2y)$.	**4**
	(c)	Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$	**7**

OR

Q.4 **(a)** Find the fixed points of $= 1 + \frac{1}{z}$. **3**

(b) Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$. **4**

(c) Solve: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **7**

Q.5 **(a)** Form the partial differential equation by eliminating arbitrary function from $z = f(x + at) + g(x - at)$. **3**

(b) Solve $p^2 + q^2 = x + y$. **4**

(c) Using method of separation of variables solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, given that $u(x, 0) = 4e^{-x}$. **7**

OR

Q.5 **(a)** Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$. **3**

(b) Solve $\frac{y^2 z}{x} p + xzq = y^2$. **4**

(c) The base of a semi-infinite strip of a metal plate is $30\ cm$ and it kept at $100^\circ\ C$. The two long edges are at zero temperature. Find the temperature at any point $15\ cm$ away from the base and situated midway between the long edges. **7**

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