

Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III EXAMINATION – SUMMER 2025

Subject Code:130001

Date:11-06-2025

Subject Name: Mathematics-III

Time:02:30 PM TO 05:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Define Beta and Gamma functions. Also prove that $\int_0^1 \frac{x dx}{\sqrt{1-x^5}} = \frac{1}{5} \beta\left(\frac{2}{5}, \frac{1}{2}\right)$. **07**

(b) Find the Fourier series expansion for $f(x) = x^2, 0 \leq x \leq 2\pi$. **07**

Q.2 (a) Obtain the Fourier series of periodic function $f(x) = 2x, -1 < x < 1, p = 2$. **07**

(b) Test for exactness and solve $[(x+1)e^x - e^y]dx - xe^y dy = 0, y(1) = 0$. **07**

OR

(b) Find the solution of differential equation $\frac{dy}{dx} + y = -\frac{x}{y}$. **07**

Q.3 (a) Solve the initial value problem $y'' + y' - 2y = 0, y(0) = 4$ and $y'(0) = -5$. **07**

(b) Find the solution of differential equation $y'' + 4y = 2\sin 3x$ by the method of undetermined co-efficients. **07**

OR

Q.3 (a) Using the method of variation of parameter, solve $y'' + y = \sec x$. **07**

(b) Solve $(D^2 + 4D + 4)y = \frac{e^{-2x}}{x^2}$. **07**

Q.4 (a) Find Laplace transforms of (i) $\sinh x \sin x$ (ii) $t e^t \sin t$ **07**

(b) Using Convolution theorem find $L^{-1}\left[\frac{1}{s(s^2+4)}\right]$ **07**

OR

Q.4 (a) (1) Find $L^{-1}\left[\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}\right]$ (2) Find $L\left\{\int_0^t e^{-u} \cos u du\right\}$ **07**

(b) Solve the equation $\frac{d^2 y}{dx^2} + y = 0$ by the power series method. **07**

Q.5 (a) A rod of length L with insulated side is initially at uniform temperature 100 . Its ends are suddenly cooled at 0°C and kept at that temperature. Find the temperature function $u(x, t)$. **07**

(b) Prove that $\int_0^\infty \frac{1-\cos\pi w}{w} \sin wx dw = \begin{cases} \frac{\pi}{2}, & \text{if } 0 < x < \pi \\ 0, & \text{if } x > \pi \end{cases}$. **07**

OR

Q.5 (a) Solve $(x^2 D^2 - 3xD + 4)y = 0, y(1) = 0, y'(1) = 3$. **07**

(b) Find Fourier cosine integral of $f(x) = e^{-kx}$, where $x > 0, k > 0$. **07**
