

Enrolment No./Seat No _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-I & II (NEW) EXAMINATION – SUMMER 2024

Subject Code:3110019

Date:05-07-2024

Subject Name:Remedial Mathematics

Time:02:30 PM TO 05:00 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Define Signum function and Logarithmic function. 03

(b) Find the limits: (a) $\lim_{x \rightarrow 1} \frac{x^2+1}{x+99}$ (b) $\lim_{x \rightarrow 1} \frac{x^{15}-1}{x^{10}-1}$ 04

(c) (i) Find $\frac{dy}{dx}$ if $y = \frac{x^5 - \cos x}{\sin x}$ 03

(ii) Solve the following system of linear equations by matrix method 04
 $x + y + z = 6, y + 3z = 11, x - 2y + z = 0$

Q.2 (a) Without expanding prove that $\begin{vmatrix} x+y & y+z & x+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$ 03

(b) Find $A.B$ if $A = \begin{bmatrix} 5 & 8 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & 0 \end{bmatrix}$ 04

(c) (i) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ 05

(ii) Derive $\sin 2x = 2 \sin x \cdot \cos x$ 02

OR

(c) (i) Find $\frac{dy}{dx}$, if $x = a \cos \theta, y = a \sin \theta$ 05

(ii) Derive the trigonometry identity: $\cos 2x = \cos^2 x - \sin^2 x$ 02

Q.3 (a) State the Quotient (division) rule of derivative and compute the derivative of $f(x) = \frac{\sin x}{\cos x}$ 03

(b) Find the distance (i) of the point $(3, -1)$ from the line $3x - 4y - 26 = 0$. 04

(ii) between two parallel lines: $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

(c) (i) Define the matrices (a) Identity matrix (b) Scalar matrix 03
(c) Diagonal matrix.

(ii) Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$ 04

OR

Q.3 (a) State the product rule of derivative and find the derivative of $f(x) = x + 3x^2 + 5x^3 + 7x^4 + 9x^5$ at $x = 1$. 03

(b) Find the point on the x-axis which is at equidistance from the point $(7,6)$ and $(3,4)$. 04

- (c) (i) If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ then find the value of x . 03
(ii) If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line. 04

Q.4 (a) Determine the order of the following differential equations 03

- (i) $\frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^3 + 2y = 0$ (ii) $\left(\frac{d^4y}{dx^4}\right)^2 + \left(\frac{d^2y}{dx^2}\right) + \sin y = 0$
(iii) $(y'')^2 + 3(y')^5 - y = 0$

(b) Solve the differential equations $ydx - (x + 2y^2)dy = 0$ 04

(c) Solve the following equations:

- (i) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ 03
(ii) $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ 04

OR

Q.4 (a) Find all points of discontinuity for the function given by $f(x) = \begin{cases} x+2 & x < 1 \\ 2 & x = 1 \\ x-2 & x > 1 \end{cases}$ 03

(b) If $y = x \cos x$ then find $\frac{d^2y}{dx^2}$ 04

(c) Solve the following equations:

- (i) $x dy = (2x^2 + 1) dx$ 03
(ii) $x \frac{dy}{dx} + 2y = x^2$ ($x \neq 0$) 04

Q.5 (a) Find the integrals: (i) $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$ (ii) $\int (4e^{3x} + \sin x + 1) dx$ 03

(b) Evaluate the integrals: (i) $\int \frac{dx}{x^2-81}$ (ii) $\int \frac{x^2-1}{x} dx$ 04

(c) (i) Evaluate $\int_1^2 \frac{1}{(x+1)(x+2)} dx$ 03

(ii) Find $\int \frac{1}{x^3-6x+13} dx$ 04

OR

Q.5 (a) State the integration by parts and find $\int x e^x dx$ 03

(b) Find $\int \frac{\sin x}{(1+\cos x)^2} dx$ 04

(c) Find $\int \frac{x^2+x+1}{(x+1)(x^2+1)} dx$ 07
