Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2024

Subject Code:2140001 **Date:15-07-2024**

Subject Name: Mathematics-4

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	Find the value of $R_e(f(z))$ and $I_m(f(z))$ for $f(z) = \frac{1}{1-z}$ at $7 + 2i$.	**03**
	(b)	Find the cube roots of unity.	**04**
	(c)	Solve the following equation by Gauss Seidel method correct up to two decimal places. $$20x + 2y + z = 30$$ $$x - 40y - 3z = -75$$ $$2x - y + 10z = 30$$	**07**
Q.2	**(a)**	Sketch the region $-1 \leq I_m(z) \leq 2$.	**03**
	(b)	Find and plot all the roots of $(1 + i)^{\frac{1}{3}}$.	**04**
	(c)	If $f(z) = u + iv$ is an analytic function of z and $u + v = x^2 - y^2 + 2xy$, find $f(z)$.	**07**
		OR	
	(c)	Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$.	**07**
Q.3	**(a)**	Determine and sketch the image of $\|z\| = 1$ under the transformation $w = z + i$.	**03**
	(b)	Determine the Mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.	**04**
	(c)	Evaluate $\int_C (x^2 - iy^2)dz$ along the parabola $y = 2x^2$ from $(1, 2)$ to $(2, 8)$.	**07**
		OR	
Q.3	**(a)**	Find the radius of convergence of the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)^{n^3} z^n$.	**03**
	(b)	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series in the interval $1 < \|z\| < 3$.	**04**
	(c)	Determine the poles of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. Hence evaluate $\int_C f(z)\, dz$, where C is the circle $\|z\| = 3$.	**07**
Q.4	**(a)**	With usual notation show that $\Delta = 1 - e^{hD}$.	**03**
	(b)	Using N-R method find an iterative formula to find q^{th} root of a positive number N.	**04**

(c) Using Gauss Elimination method solve the following system of equations. **07**

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 20$$

OR

Q.4 **(a)** Prove that $\oint_C \frac{dz}{z-a} = 2\pi i$, where C is the circle $|z-a| = r$. **03**

(b) Find the value of y for $x = 21$ from the following data. **04**

x:	20	23	26	29
y:	0.3420	0.3907	0.4384	0.4848

(c) Using Runge-Kutta method of fourth order to find $y(0.1), y(0.2)$ given $\frac{dy}{dx} = 2x + y, y(0) = 1$. **07**

Q.5 **(a)** Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using trapezoidal rule with $h = 0.2$. **03**

(b) Using Langrange's interpolation formula fit a polynomial to the given data: **04**

x:	0	1	2	5
$f(x)$:	2	3	12	147

(c) Using Taylor's series method, find correct to four decimal place the value at $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$. **07**

OR

Q.5 **(a)** Evaluate $\int_0^3 \frac{dx}{1+x}$ with $n = 6$, by using Simpson's $\frac{3}{8}$ rule. **03**

(b) Apply stirling formula to find $y(35)$ from th following data: **04**

x:	20	30	40	50
y:	512	439	346	243

(c) Using Secant method find the real root of the equation $xe^x - 1 = 0$ correct up to three decimal places between 0 and 1. **07**

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