

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER-I & II(NEW) EXAMINATION – SUMMER 2023

Subject Code:3110019

Date:10-08-2023

Subject Name:Remedial Mathematics

Time:10:30 AM TO 01:00 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Evaluate the determinant $A = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$. 03

(b) Find $A.B$ if $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}$. 04

(c) (1) If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ find a, b, c, x, y and z . 03

(2) Solve the following system of equations by matrix method. 04
 $2x + 5y = 1$
 $3x + 2y = 7$

Q.2 (a) Find distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ 03

(b) If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line. 04

(c) (1) Define modulus function and greatest integer function. 03
 (2) Deduce trigonometric identity $\sin 3x = 3\sin x - 4\sin^3 x$ 04

OR

(c) (1) Define exponential function and logarithmic function 03
 (2) Deduce trigonometric identity $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$, $x \neq n\pi + \frac{\pi}{2}$, $n \in \mathbb{Z}$. 04

Q.3 (a) Evaluate: $\lim_{x \rightarrow 2} \left(\frac{x^3 - 4x^2 + 4x}{x^2 - 4} \right)$ 03

(b) Find $\frac{dy}{dx}$, if $y = \frac{x^5 - \cos x}{\sin x}$ 04

(c) (1) Find derivative of $y = \sin(x^2)$ 03
 (2) If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$. 04

OR

Q.3 (a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x} \right)$ 03

(b) Find $\frac{dy}{dx}$, if $x = a \cos \theta$, $y = a \sin \theta$ 04

(c) (1) Find derivative of $y = \cos^{-1}(e^x)$. 03

(2) Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$. 04

Q.4 (a) Find all points of discontinuity for the function given by 03

$$f(x) = \begin{cases} x+2 & x < 1 \\ 0 & x = 1 \\ x-2 & x > 1 \end{cases}$$

(b) Find order and degree of following differential equations. 04

(1) $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$

(2) $\left(\frac{d^3y}{dx^3} \right)^2 + \left(\frac{d^2y}{dx^2} \right)^3 + \left(\frac{dy}{dx} \right)^4 + y^5 = 0$

(c) Solve the following.

(1) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ 03

(2) $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$. 04

OR

Q.4 (a) Discuss the continuity of $f(x) = \begin{cases} x^3 + 1 & x \neq 0 \\ 1 & x = 0 \end{cases}$ at $x = 0$. 03

(b) Find $\frac{dy}{dx}$ if, 04

(1) $y + \sin y = \cos x$

(2) $x - y = \pi$

(c) Solve the following.

(1) $x dy = (2x^2 + 1) dx$ 03

(2) $x \frac{dy}{dx} + 2y = x^2 \ (x \neq 0)$ 04

Q.5 (a) State integration by parts and find $\int x e^x dx$ 03

(b) Find the following integrals. 04

(1) $\int \frac{x^3-1}{x^2} dx$

(2) $\int \left(x^{\frac{2}{3}} + 1 \right) dx$

(c) Evaluate followings.

(1) $\int_1^2 \frac{1}{(x+1)(x+2)} dx$ 03

(2) $\int_1^2 \frac{\tan^{-1} x}{1+x^2} dx$ 04

OR

Q.5 (a) Find $\int \frac{1-\sin x}{\cos^2 x} dx$ 03

(b) Find $\int \frac{1}{x^3-6x+13} dx$ 04

(c) Find $\int \frac{x^2+x+1}{(x+1)(x^2+1)} dx$ 07
